# **Three Point Bending of Seven Layer Beams**

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**Abstract.** The subject of the paper is a symmetrical seven layer beam with foam cores. The beam consists of the main core, two inner sheets, two outer cores and two external sheets. The cores are metal foams, while the sheets are metal. The analytical model of the beam is developed. The displacement is formulated with consideration of the zig-zag hypothesis of deformation of a flat cross-section of the beam and the governing differential equations for the considered beam are derived based on the stationary total potential energy. The studies are devoted to deflections of the beams under a three point bending. The influence of the different types of cores and faces on the deflections of the beam is analyzed. Moreover, the numerical FEM-model of the beam is presented in two different FEM programs: ABAQUS System and SolidWorks Simulation. The results of the analysis are presented in Tables and Figures.

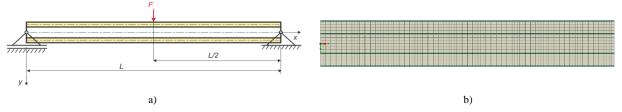
## INTRODUCTION

A generalization of classical sandwich structures involves, for example, multilayer structures. Carrera [1] presented a review of the theories (including zig-zag theories) for the analysis of multilayered structures while Chakrabarti *et al* [2] described higher order zig-zag theory for the static analysis of laminated sandwich beam with soft core. Magnucka-Blandzi and Magnucki [3] and Magnucka-Blandzi [4] analyzed a simply supported sandwich beam with a metal foam core using different hypotheses of deformation of a plane cross section of the beam. Paczos *et al* [5] presented an experimental and numerical analysis of orthotropic five-layers sandwich beams. Magnucka-Blandzi and Rodak [6] studied deflection and critical axial force for seven layer beams with trapezoidal corrugated cores.

The objectives of the study is comparison of the analytical and FEM results of deflection values of the seven layers beam with foam cores (Fig 1).

### ANALYTICAL AND NUMERICAL MODEL OF THE BEAM

The subject of the study is a simply supported symmetrical seven layer beam of length L and width b.



**FIGURE 1.** Scheme of the symmetrical seven layer beam under load in three point bending a) analytical model, b) numerical model of a quarter of the beam

The beam consists of the main core of thickness  $t_{c1}$  and Young's modulus  $E_{c1}$ , two inner sheets of thicknesses  $t_s$  and Young's modules  $E_{si}$ , two second cores of thicknesses  $t_{c2}$  and Young's modules  $E_{c2}$ , and two outer sheets of thicknesses  $t_s$  and Young's modules  $E_{so}$ . The main core and two face cores are metal foams, while the sheets are metal.

The studies are carried out for example beams of width b=10 mm, total thickness  $h = t_{c1} + 2 t_{c2} + 4 t_s$  (h=40 mm), where thicknesses  $t_{c1}=16$  mm,  $t_{c2}=10$  mm,  $t_{s}=1$  mm and force F=2 kN. The values of material constants of beam layers are specified in Table 1.

**TABLE 1.** The values of Young's modules E[MPa] and Poisson ratios v

Material	Steel	Steel Foam	Aluminum	Al Foam
E [MPa]	200000	3000	79000	220
$\nu$	0.3	0.05	0.33	0.3

The three beams are chosen to the studies and they are characterized by various materials of the layers (Table 2).

**TABLE 2.** The three type beams with various materials of the layers

Beam	Main core - tc1	Inner sheets - t <sub>s</sub>	Second cores $-t_{c2}$	Outer sheets - ts
B-1	Steel foam	Steel	Steel foam	Steel
B-2	Al foam	Aluminum	Steel foam	Steel
B-3	Al foam	Aluminum	Al foam	Aluminum

The same numerical model of the 1/4 of the beam is considered in ABAQUS system and SolidWorks Simulation program to compare obtained results with each other and analytical solutions.

The results of maximal analytical and numerical deflection are specified in Table 3, where  $v_{max}^{(FEM\_A)}$  and  $v_{max}^{(FEM\_S)}$  are deflections calculated in ABAQUS system and SolidWorks Simulation program, respectively.

TABLE 3. The values of maximal deflections of beams – analytical and FEM

	L/I	L/h=6 (L=240 mm)		L/h=10 (L=400 mm)		L/h=14 (L=560 mm)			
Beam	B-1	B-2	B-3	B-1	B-2	B-3	B-1	B-2	B-3
$v_{max}^{(Analyt)}[mm]$	0.4845	1.5420	3.8887	1.6987	3.9984	9.0960	4.2302	7.9238	17.8376
$v_{max}^{(FEM\_A)}[mm]$	0.5060	1.6134	4.1971	1.7192	4.0785	9.4069	4.2479	8.0037	18.1495
$v_{max}^{(FEM\_S)}[mm]$	0.4914	1.6755	4.1914	1.6944	4.1298	9.4018	4.2086	8.0435	18.1455

## **CONCLUSIONS**

The proposed objectives of the study are achieved. The analytical and numerical results presented in Table 3 allow to conclude that the relative differences between the deflection values obtained with analytical and numerical methods do not exceed 8.7 per cent. Additionally, the zig-zag theory is recommended in order to model the multilayered structures.

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